

**What Is Claimed Is:**

- 1           1.       A method for bounding the solution set of a system of linear  
2 equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an interval vector, the  
3 method comprising:  
4           preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through  
5 by a matrix  $\mathbf{B}$  to produce a preconditioned set of linear equations  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ,  
6 wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$ ;  
7           widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
8 midpoints of the elements of  $\mathbf{M}$  form the identity matrix; and  
9           using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds  
10 the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ .
- 1           2.       The method of claim 1, wherein the method further comprises  
2 computing the matrix  $\mathbf{B}$  by:  
3           computing an approximate center  $\mathbf{A}_C$  of the matrix  $\mathbf{A}$ ; and  
4           forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .
- 1           3.       The method of claim 1, wherein using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  
2  $\mathbf{h}$  involves:  
3           forming  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ;  
4           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6 which the  $i$ -th element is 1 and other elements are 0;  
7           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
8           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and  
9           setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           4.       The method of claim 1, further comprising assuring that  $\sup(r_i) \geq 0$   
2       by changing the sign of  $r_i$  (and  $x_i$ ) if necessary.

1           5.       The method of claim 1, further comprising:  
2       determining if  $\mathbf{M}$  is regular; and  
3       using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
4       regular.

1           6.       A computer-readable storage medium storing instructions that  
2       when executed by a computer cause the computer to perform a method for  
3       bounding the solution set of a system of linear equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an  
4       interval matrix and  $\mathbf{b}$  is an interval vector, the method comprising:  
5       preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through  
6       by a matrix  $\mathbf{B}$  to produce a preconditioned set of linear equations  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ,  
7       wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$ ;  
8       widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
9       midpoints of the elements of  $\mathbf{M}$  form the identity matrix; and  
10       using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds  
11       the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ .

1           7.       The computer-readable storage medium of claim 6, wherein the  
2       method further comprises computing the matrix  $\mathbf{B}$  by:  
3       computing an approximate center  $\mathbf{A}_C$  of the matrix  $\mathbf{A}$ ; and  
4       forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           8.       The computer-readable storage medium of claim 6, wherein using  
2 **M** and **r** to compute the hull **h** involves:  
3           forming **P** as an inverse of the left endpoint of **M**;  
4           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6 which the  $i$ -th element is 1 and other elements are 0;  
7           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
8           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and  
9           setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           9.       The computer-readable storage medium of claim 6, wherein the  
2 method further comprises assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$   
3 (and  $x_i$ ) if necessary.

1           10.      The computer-readable storage medium of claim 6, wherein the  
2 method further comprises:  
3           determining if **M** is regular; and  
4           using the Gauss-Seidel process for computing the hull **h** if **M** is not  
5 regular.

1           11.      An apparatus that bounds the solution set of a system of linear  
2 equations  $\mathbf{Ax} = \mathbf{b}$ , wherein **A** is an interval matrix and **b** is an interval vector,  
3 comprising:  
4           a preconditioning mechanism that is configured to precondition the set of  
5 linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through by a matrix **B** to produce a  
6 preconditioned set of linear equations  $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$ ;



2025-04-10 14:49:44

1           15.    The apparatus of claim 11, wherein the preconditioning mechanism  
2 is configured to:  
3           determine if  $\mathbf{M}$  is regular; and to  
4           terminate the process of computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not regular.

1           16.    A method for bounding the solution set of a system of linear  
2 equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through by the matrix  $\mathbf{B}$  to produce a  
3 preconditioned set of linear equations  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$ , the  
4 method comprising:  
5           assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$  (and  $x_i$ ) if necessary;  
6           widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
7 midpoints of the elements of  $\mathbf{M}$  form the identity matrix; and  
8           using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds  
9 the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$  by,  
10           forming  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ,  
11           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ,  
12           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a  
13 unit vector in which the  $i$ -th element is 1 and other elements are 0,  
14           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ,  
15           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ , and  
16           setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           17.    The method of claim 16, further comprising:  
2           determining if  $\mathbf{M}$  is regular; and  
3           using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
4 regular.

- 1           18.    The method of claim 16, wherein the method further comprises  
2    computing the matrix **B** by:  
3            computing an approximate center  $A_C$  of the matrix **A**; and  
4            forming **B** by computing an approximate inverse of  $A_C$ ,  $B = (A_C)^{-1}$ .